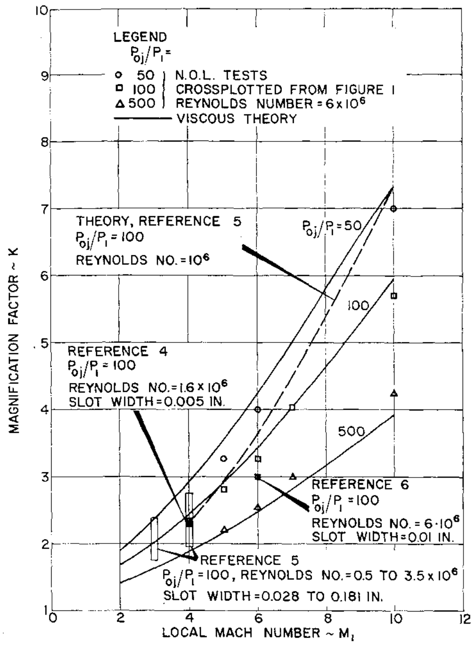

Fig. 5 Comparison of theoretical and experimental magnification factors (turbulent boundary layer).

An investigation of Reynolds number effects at Mach 6 was conducted, and these results are shown in Fig. 2. Although, as will be shown later, increased Reynolds number is expected to slightly reduce effectiveness, the peaking of the high Reynolds number data was not anticipated, and no theoretical explanation of this effect is known to the author.

Theoretical Analysis

An analytical expression for the sonic slot magnification factor, taken from Ref. 1, is

$$K = 1 + \frac{(\Delta P/p_1\delta)(h/w)}{2[2/(\gamma + 1)]^{i/(\gamma - i)}(P_{0i}/P_1) - 1} \tag{1}$$

Utilizing the turbulent boundary-layer plateau pressure rise correlation presented in Ref. 4, together with an equivalent wedge angle δ required to produce the pressure rise, the term $\Delta P/P_1\delta$ becomes

$$\frac{\Delta P}{P_1\delta} = 2.6\,\frac{(M_1{}^2 - 1^{1/2})}{(R_1)^{1/10}\delta}\ \ \mathrm{rad}^{-1} \tag{2}$$

This expression is plotted in Fig. 3.

It was assumed in Ref. 1 that the jet penetration height h was equivalent to the area ratio required to isentropically expand the jet flow to ambient pressure. However, schlieren observations of this study and of Ref. 5 have shown the actual penetration heights to be approximately twice as great for a given jet pressure ratio. These data are shown in Fig. 4 and provide the basis for evaluation of the h/w term in Eq. (1).

Figure 5 compares this theoretical prediction and the theory of Ref. 5 with the experimental data shown in Fig. 1 and data of Refs. 4–6. Apparently the linear addition of viscous and inviscid effects, as proposed in Ref. 5, is overly optimistic at hypersonic Mach numbers, whereas the viscous theory originally proposed in Ref. 1 appears to properly indicate Mach number and jet pressure ratio effects. Further experimental investigation of Reynolds number effects on reaction control performance is required.

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Convection from Heated Wires at Moderate and Low Reynolds Numbers

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TWO-dimensional convection from heated wires placed transverse to a stream is of interest in hot-wire anemometry. The correlations that have been presented by King,[1] Hilpert,[2] McAdams,[3] and Collis and Williams[4] are particularly well known. Each of the correlations given by these authors is chosen because it is found empirically to fit the relevant measurements for a limited range of Reynolds number. None of the correlation equations has been chosen because it explicitly represents detailed knowledge of the local flow around the surface of a cylinder and the consequent local distribution of heat-transfer coefficient. Recently, correlations have been proposed[5] for convection from cylinders at high Reynolds numbers which do attempt to take explicit account of the local variation of heat transfer around the cylinder. The purpose of this short paper is to show that the latter correlation can be used to represent heat transfer from wires at moderate and low Reynolds numbers down to a Reynolds number of about 1.0 when certain slight adaptations are made to it to represent some of the physical changes in the flow pattern as the Reynolds number becomes smaller. The availability of a single correlation, covering a large range of Reynolds number, can make the interpretation of hot-wire measurements an easier and more straightforward process.

At moderate and low Reynolds numbers it is apparent that freestream turbulence does not have a strong influence upon the heat transfer on the forward portion of a cylinder, in contrast with the findings at high Reynolds numbers. For this reason the discussion here will be centered upon the correlation given[5] for heat transfer at high Reynolds number but low turbulence intensities:

$$Nu = 0.37\ Re^{1/2} + 0.057\ Re^{2/3} \tag{1}$$

In this equation, the first term represents the heat transfer from the forward portion of the cylinder (which is covered by a laminar boundary layer), while the second term represents

Received September 8, 1964. This study was made in connection with research into separated flows supported by National Science Foundation Grant GP-978.

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the heat transfer from the region under separated flow. The laminar boundary-layer analysis upon which the numerical value of the coefficient of the first term is based is an asymptotic analysis, valid when the boundary-layer thickness is very small compared with the radius of the cylinder. For moderate Reynolds numbers, it becomes necessary to correct the numerical value of the coefficient to account for the finite thickness of the laminar boundary layer with respect to the cylinder radius. In order to obtain a reasonable estimate for this correction, it appears suitable to adapt an idea once proposed by Langmuir[6] in connection with the analysis of natural convection from a heated horizontal cylinder. In the asymptotic analysis, it is assumed that the boundary layer is locally a flat film. In the vicinity of the cylinder surface, the heat transfer occurs predominantly by pure conduction in the gas, the net convection velocities being very small. With this in mind, it appears reasonable to suppose that, for finite Reynolds numbers, the conduction occurs in a thick-walled "tube" of the gas, the "tube" thickness being proportional to the boundary-layer thickness. A first-order estimate for the conduction thickness can be obtained from the asymptotic solution. If the heat transfer were indeed by conduction through the gas, the corresponding wall thickness of a plane slab is given by the reciprocal of the slope of the temperature profile at the wall surface. For the forward stagnation region, the corresponding conduction thickness is $1.01\,D/(Re_D)^{1/2}$. In the steady conduction problem, which corresponds to the situation at moderate Reynolds number, a logarithmic expression arises. The logarithm can be expanded as a series (the radius of convergence of which embraces all Reynolds numbers exceeding 4.04), and the leading term of the series can be used in a correction factor. The laminar boundary-layer thickness around the forward portion of a cylinder is roughly constant, so that, to the order of approximation which is involved, the correction factor calculated for the forward stagnation point can be applied to the term in the equation which represents the whole of the forward region on the cylinder. This equation therefore becomes

$$\begin{aligned} Nu &= 0.37\,Re^{1/2}\,\{1 + 1.01/Re^{1/2}\} + 0.057\,Re^{2/3} \\ &= 0.3737 + 0.37\,Re^{1/2} + 0.057\,Re^{2/3} \end{aligned} \qquad (2)$$

Since the contribution to the total heat transfer of the forward portion of the cylinder is greater than that of the rear portion, at moderate Reynolds numbers, it is appropriate that the term for the forward region is the first to undergo correction. As the Reynolds numbers considered become smaller, it is to be expected that it is necessary to take account of more terms in the series, to take account of the changing position of the separation of the laminar boundary layer, and of changes in the heat-transfer characteristics of the rearward surface. It seems that, to some extent, corrections for these various factors compensate each other, and it is not surprising that Eq. (2) correlates heat-transfer measurements down to Reynolds numbers below those for which the flow models used to obtain the equation are valid.

Comparison of Eq. (2) with the correlation of McAdams shows good agreement for Reynolds numbers as low as 1.0. Collis and Williams presented two equations to correlate their own measurements, the first being for $0.02 < Re < 44.0$ and the second for $44.0 < Re < 140$. Equation (2), when compared with these equations, shows differences ranging from 0 to 6%, with the largest differences occurring at $Re = 44.0$, for the range $1.0 < Re < 140$. The differences becomes more uniform and of the order of 3–4% if the coefficient of the first term in the equation is changed to 0.36 instead of 0.37.

It may be concluded that Eq. (2) represents convection from heated cylindrical surfaces within experimental uncertainty over a range of Reynolds number of about 1.0 to 10^5, subject to the conditions that, at high Reynolds numbers, the relation is appropriate for streams with low turbulence intensity and, for all Reynolds numbers, the Grashof numbers are not significant. The correlation is based upon a physical understanding of the local heat-transfer distribution appropriate to high and moderate Reynolds numbers; it is a fortunate but accidental circumstance that the equation provides a good correlation for Reynolds numbers down to the order of unity. The correlation is for heat transfer that occurs with small temperature differences; when the temperature differences become large, it is necessary to include a factor to account for this, as is done by Collis and Williams. As a secondary matter, the correlation given here provides a hint of the reasons for which the empirical correlations having the form

$$Nu = A + B\,Re^n \qquad (3)$$

work as well as they do. It may be noted from Eq. (2) that a numerical constant A arises from the first-order correction of the laminar convection on the forward portion of the cylindrical body. When the range of Reynolds numbers used in a correlation is small, there is considerable latitude in the choice of the coefficients in fitting the data.

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Near-Equilibrium Criterion for Complex Reacting Flows

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Introduction

A near-equilibrium criterion is required when determining the point in a nozzle where integration of an initially equilibrium flow should be started. A similar criterion is required if a "sudden-freezing" analysis is to be performed. Although it is easy to examine the proximity to equilibrium of a simple reacting system (a dissociating gas, for example), the problem is more complicated in the general case when several reactions may be simultaneously taking place among several species.[1]

An examination of the closeness to equilibrium can be made from a comparison of the characteristic reaction time with the characteristic flow time. This method was first developed by Penner[2] for flow with a single reaction and extended by Bray and Appleton[3] to an arbitrary number of reactions. When applied to complex systems, it suffers from the disadvantage that each reaction is considered independently. No account is taken of the coupling between reactions whereby a slow reaction can be held close to equilibrium by the action of a set of fast reactions.

Received September 11, 1964.

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